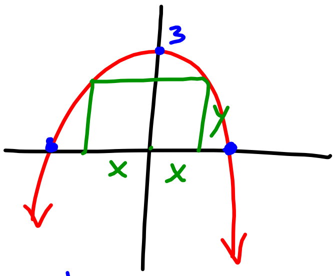
# 4/9/18

1. What are the dimensions that maximize the area of a rectangle having two lower corners on the x-axis and the two upper corners on the graph of $y = 3 - x^2$? What is the maximum area?




$A = 2xy$

$A = 2x(3 - x^2)$

$A = 6x - 2x^3$

$A' = 6 - 6x^2$

$0 = 6 - 6x^2$

$6x^2 = 6$

$x^2 = 1$

$x = \pm 1$

rest:

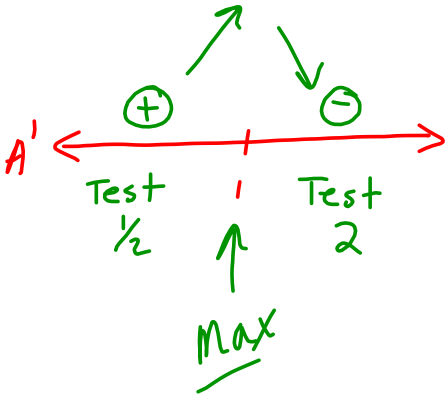
$0 < y < 3$

$0 < x < \sqrt{3}$

$0 = 3 - x^2$

$x^2 = 3$

$x = \pm\sqrt{3}$




$x = 1$

$y = 3 - (1)^2$

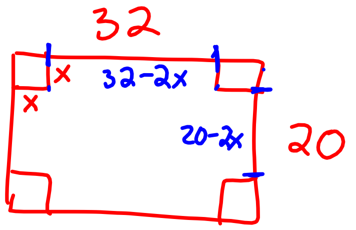
$y = 3 - 1$

$y = 2$

Max Area $= 2(1)(2)$

$= \boxed{4 \text{ units}^2}$

2) A rectangular box, open on the top, is to be constructed from a 20in. by 32in. piece of sheet metal by cutting identical squares from each of the corners and folding up the flaps. What is the length of the sides of the squares that will maximize the volume of the box? What is the maximum volume of the box?




$V = LWH$

$V = (32-2x)(20-2x)(x)$

$V = 640x - 104x^2 + 4x^3$

Rest:
$0 < x < 10$

$V' = 640 - 208x + 12x^2$

$V' = 12x^2 - 208x + 640$

$\frac{0}{4} = \frac{12x^2 - 208x + 640}{4}$

$0 = 3x^2 - 52x + 160$

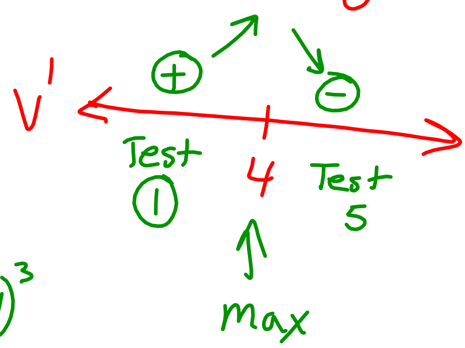
$x = \frac{52 \pm \sqrt{(-52)^2 - 4(3)(160)}}{2(3)}$

$x = \frac{52 \pm \sqrt{784}}{6}$

$x = \frac{52 \pm 28}{6}$

$\frac{52+28}{6} = \frac{80}{6} = \cancel{13.\overline{3}}$ reject

$\frac{52-28}{6} = \frac{24}{6} = 4$



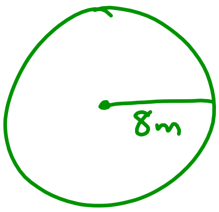

$\boxed{\text{Sides of } \square \text{ are } 4 \text{ in}}$

Volume =

$V = 640(4) - 104(4)^2 + 4(4)^3$

$\boxed{V = 1152 \text{ in}^3}$

3) A Stone thrown into a pond produces a circular ripple which expands from the point of impact. If the radius of the ripple increases at the rate of 1.5m/sec, how fast is the disturbed area growing when the radius is 8m?




Know

$\frac{dr}{dt} = 1.5$

$r = 8$

Need

$\frac{dA}{dt}$

Formula

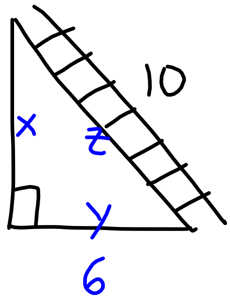
$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(8)(1.5)$

$\boxed{\frac{dA}{dt} = 24\pi \frac{m^2}{sec}}$

4) A 10ft ladder is leaning against a wall. The foot of the ladder is sliding away from the wall at 2ft/sec. How fast is the top of the ladder sliding down the wall when the foot of the ladder is 6 feet from the wall?




Know:

$x = 8$

$y = 6$

$z = 10$

$\frac{dy}{dt} = 2 \text{ ft/s}$

$\frac{dz}{dt} = 0$

(ladder doesnt change)

Need:

$\frac{dx}{dt}$

$10^2 = 6^2 + x^2$

$100 = 36 + x^2$

$64 = x^2$

$8 = x$

Equation:

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$2(8)\frac{dx}{dt} + 2(6)(2) = 2(10)(0)$$

$$16\frac{dx}{dt} + 24 = 0$$

$$16\frac{dx}{dt} = -24$$

$$\frac{dx}{dt} = \frac{-24}{16} = -\frac{3}{2} \text{ ft/s}$$

Ladder slides down @ $\frac{3}{2}$ ft/sec

5) If $f''(x) = 6x^2 - 12x + 2$, $f'(1) = -3$, and $f(-2) = 1$, find $f(x)$.

$$f'(x) = \frac{6x^3}{3} - \frac{12x^2}{2} + \frac{2x}{1} + c$$

$$f'(x) = 2x^3 - 6x^2 + 2x + c$$

$$-3 = 2(1)^3 - 6(1)^2 + 2(1) + c$$

$$-3 = 2 - 6 + 2 + c$$

$$-3 = -2 + c$$

$$-1 = c$$

$$f'(x) = 2x^3 - 6x^2 + 2x - 1$$

$$f(x) = \frac{2x^4}{4} - \frac{6x^3}{3} + \frac{2x^2}{2} - 1x + d$$

$$f(x) = \frac{1}{2}x^4 - 2x^3 + x^2 - 1x + d$$

$$1 = \frac{1}{2}(-2)^4 - 2(-2)^3 + (-2)^2 - 1(-2) + d$$

$$1 = \frac{1}{2}(16) + 16 + 4 + 2$$

$$1 = 8 + 16 + 4 + 2 + d$$

$$1 = 30 + d$$

$$-29 = d$$

$$\boxed{f(x) = \frac{1}{2}x^4 - 2x^3 + x^2 - 1x - 29}$$

6) $\int 3x^2 + 6x + 5 \, dx$

$= \frac{3x^3}{3} + \frac{6x^2}{2} + 5x + c$

$= x^3 + 3x^2 + 5x + c$

7) $$\int_2^6 2x^4 - 3x^3 + 1x^2 - 5x + 14 \; dx$$

$$= \frac{2x^5}{5} - \frac{3x^4}{4} + \frac{1x^3}{3} - \frac{5x^2}{2} + 14x + c \Big]_2^6$$

$$= \frac{2(6)^5}{5} - \frac{3(6)^4}{4} + \frac{6^3}{3} - \frac{5(6)^2}{2} + 14(6)$$

$$- \left(\frac{2(2)^5}{5} - \frac{3(2)^4}{4} + \frac{2^3}{3} - \frac{5(2)^2}{2} + 14(2)\right)$$

$$= \boxed{2182.933}$$